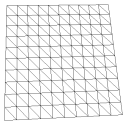
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Modeling Deformable Surfaces from Single Videos

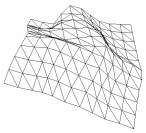
CVLab, EPFL, Switzerland
http://cvlab.epfl.ch/

Monocular 3D Reconstruction

Reference



Input



Problem Definition

Input:

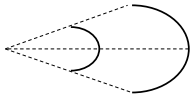
- Reference image.
- Corresponding 3D surface.
- Projection matrix P.
- 3D-to-2D correspondences between reference configuration and input image.

Unknowns:

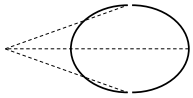
- Mesh vertex coordinates corresponding to input image

$$\mathbf{X} = [x_1, y_1, z_1, \cdots, x_{n_v}, y_{n_v}, z_{n_v}]^T$$

Ambiguity



Scale ambiguity



Bas-Relief Ambiguity

- 3D Shape or deformation models are needed.

➢ How can we design models that do not make unwarranted assumptions?

Talk Outline

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.

Linear Formulation

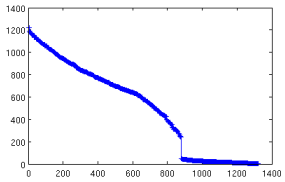
- Calibrated camera, **A** intrinsic parameters matrix.
- Coordinates expressed in the camera referential.
- Unknown mesh vertex coordinates: $\mathbf{X} = (\mathbf{v}_1^T, ..., \mathbf{v}_{n_v}^T),\ \mathbf{v}_i = (x_i, y_i, z_i)^T$
- Correspondences $(u_i, v_i)^T$
  - Barycentric coordinates from reference configuration: (a_i, b_i, c_i)
  - Current image location:
- Correspondence equation:

$$\begin{pmatrix} u_i \\ v_i \\ 1 \end{pmatrix} = \frac{1}{k_i} \mathbf{A}(a_i \mathbf{v}_1 + b_i \mathbf{v}_2 + c_i \mathbf{v}_3)$$

Salzmann et al., CVPR 2007

Linear System and Singular Values

- Solve: $\mathbf{MX} = 0$



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- Potential applications.

Inextensible Meshes

A solution of the linear system belongs to the kernel of $\mathbf{M}$:

$$\mathbf{MX} = 0 \Rightarrow \mathbf{X} = \sum_i \beta_i \mathbf{p}_i ,$$

where the $\mathbf{p}_i$ are the eigenvectors corresponding to small eigenvalues.

Inextensible mesh:

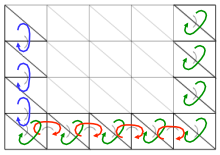
$$\left\| \sum_i \beta_i \mathbf{p}_i^{\,j} - \sum_i \beta_i \mathbf{p}_i^{\,k} \right\|^2 = \text{cte}$$

for all neighboring vertices j and k.

--> A system a quadratic equations that could be solved in closed form using extended linearization, but with too many variables for existing solvers.

Salzmann et al., ECCV 2008

Dimensionality Reduction



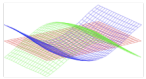
Database of Feasible Shapes

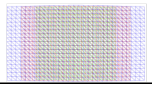
PCA

$$\mathbf{X} = \mathbf{X}_0 + \sum_i \alpha_i \mathbf{S}_i$$

$$= \mathbf{X}_0 + \mathbf{SA}$$

$$\text{with } \mathbf{A} = \begin{bmatrix} \alpha_1 & \dots & \alpha_N \end{bmatrix}^T$$

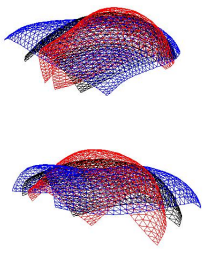


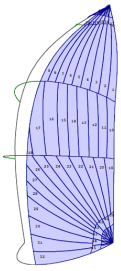


Salzmann et al., PAMI 2007

Alinghi

Spinnaker Modes



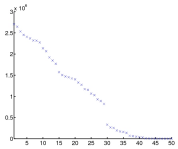


Reduced System

$$\mathbf{MX} = 0$$

$$\mathbf{X} = \mathbf{X}_0 + \mathbf{SA}$$

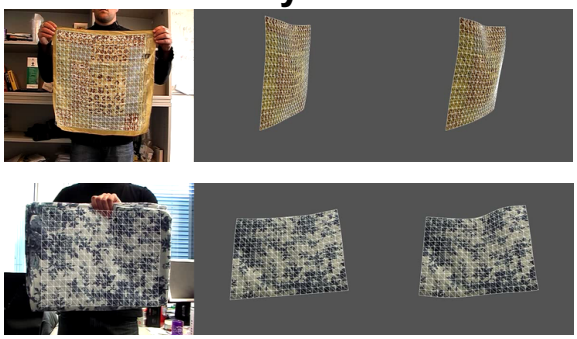
$$\Rightarrow \begin{bmatrix} \mathbf{MS} & \mathbf{MX}_0 \\ \mathbf{W} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ 1 \end{bmatrix} = 0$$



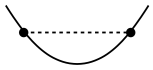
where the **W** is a diagonal matrix of modal penalty terms that depends on the eigenvalues of the training data covariance matrix.

- **A** can also be written as a weighted sum of eigenvectors of the extended matrix.
- The inextensibility constraints give rise to a smaller set of quadratic equations than can now be solved.

Independent Detection in Every Frame



Limitation



In the presence of sharp folds:

- The Euclidean distance between discrete points decreases.
- Inextensibility constraints are not appropriate anymore.

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Talk Outline

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
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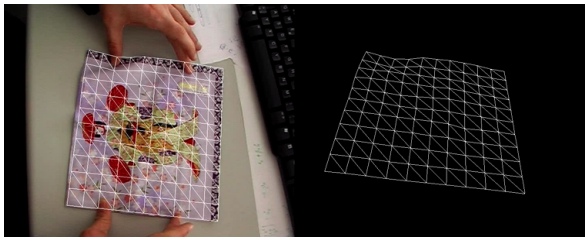
- Potential applications

Handling Creases



Replace inextensibility constraints by distance inequalities that:

- Let us reconstruct surfaces with sharp folds.
- Yield a convex formulation of the reconstruction problem.

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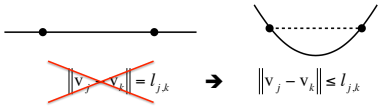
Salzmann et al., CVPR 2009

Inequality Constraints

In the presence of sharp folds, geodesic distances remain constant, but Euclidean ones may decrease.



Naive formulation:

$$\mathbf{X}_{opt} = \arg\min \|\mathbf{MX}\| ,$$

subject to

$$\|\mathbf{v}_j - \mathbf{v}_k\| \leq d_{jk}$$

for all neighboring vertices j and k.

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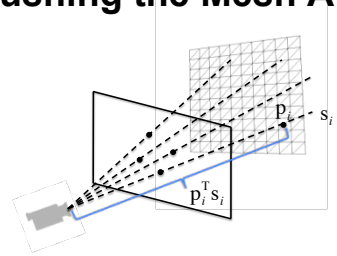
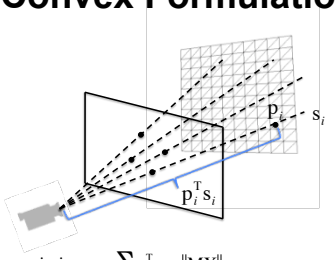
Pushing the Mesh Away



- Inequality constraints do not prevent the mesh from shrinking.
- To this end, we push the points along their lines-of-sight as far as the constraints allow.

Convex Formulation



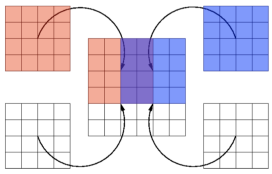
$$\underset{X}{\text{maximize}} \quad w_d \sum_{i \in N_c} p_i^T s_i - \|MX\|$$

$$\text{subject to} \quad \|v_j - v_k\| \leq l_{j,k} \;, \quad \forall (j,k) \in \text{Edges}$$

This is an SOCP problem, which can be solved using standard numerical routines.

Shape Regularization

- Regularization is needed to enforce smoothness on poorly textured parts.
- To handle sharp folds, the global models must be replaced by local ones.



→ Introduce a linear model for individual surfaces patches

$$X^i = X_0^i + \Lambda c^i$$

Local Deformation Model

- To avoid having to explicitly force the coefficients of overlapping patches to be consistent, we express them as

$$c^i = \Lambda^T (X^i - X_0^i) \;,$$

which arises from the orthonormality of the modes.

- Regularization is achieved by penalizing the coefficients associated to high energy modes, which is done by minimizing

$$\sum_{i \in N_p} w_i \left\| \Sigma^{-1/2} \Lambda^T (X^i - X_0^i) \right\| ,$$

where Σ contains the eigenvalues of the training data covariance matrix.

Temporal Regularization

For short video-sequences, we can enforce temporal consistency by introducing a second order---constant speed---motion model:

$$\Delta_{t-1,t} = \Delta_{t,t+1}$$

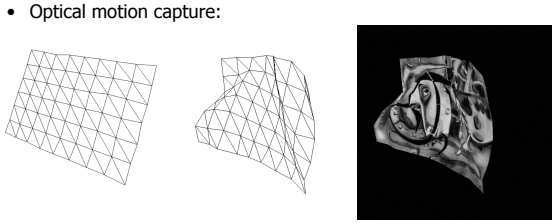
$$X^{t-1} \quad X^t \quad X^{t+1}$$

→ We solve our optimization problem for 3 frames simultaneously, and regularize the motion between frames by minimizing

$$\left\| X^{t-1} - 2X^t + X^{t+1} \right\| .$$

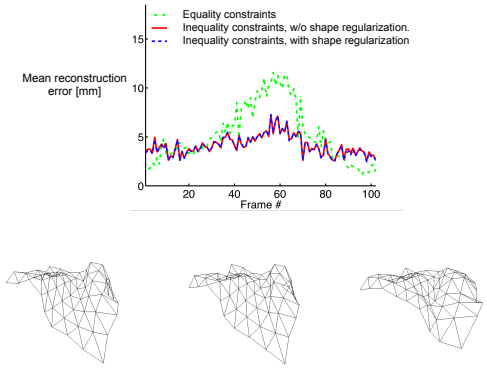
Synthetic Data

- Optical motion capture:



- Correspondences:
  - Sample the barycentric coordinates, project the 3D points, add Gaussian noise with variance 5 to the image locations.
  - Compute SIFT matches between the input images and the reference.

Synthetic Correspondences



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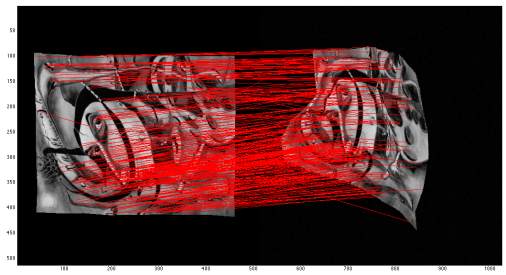
SIFT Correspondences



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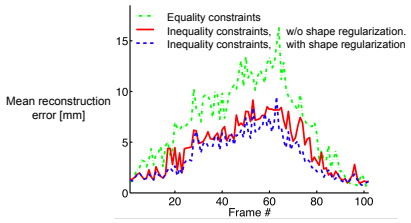
SIFT Correspondences



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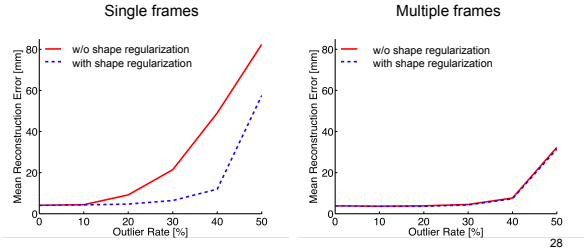
Introducing Outliers

- Synthetic correspondences.
- Varying outlier rate.



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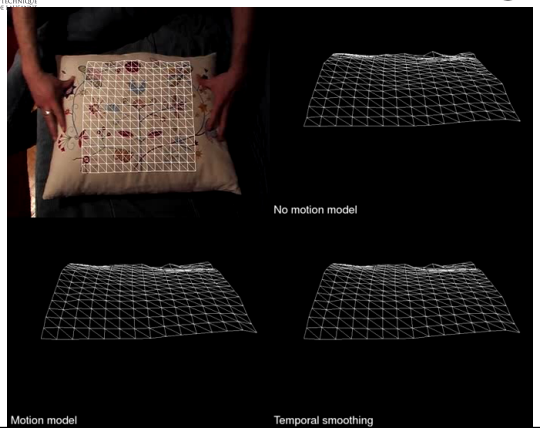
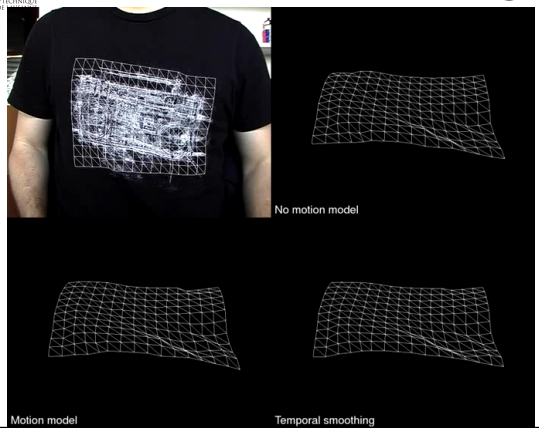
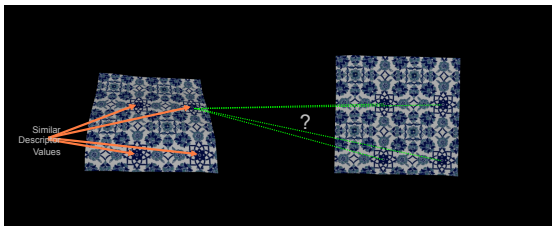




Repetitive Patterns



Problem:
- Correspondences are difficult to establish.

Solution:
- Simultaneously solve for correspondences and 3D shape.

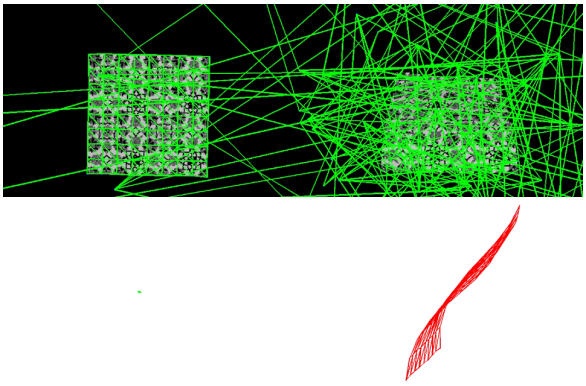
Shaji et al., CVPR 2010

Mixed Integer Quadratic Problem

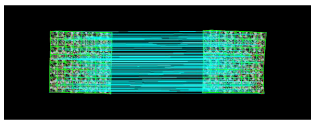
$$\min_{\text{matches,reconstruction}} \left(\underbrace{\text{E(matching score)}}_{\textit{binary terms}} + \underbrace{\text{E(match(Reconstruction Cost))}}_{\textit{mixed terms}} \right)$$

- Instance of a NP Hard Problem.
- Branch-and-bound methods that works well for this particular problem.

Iterations

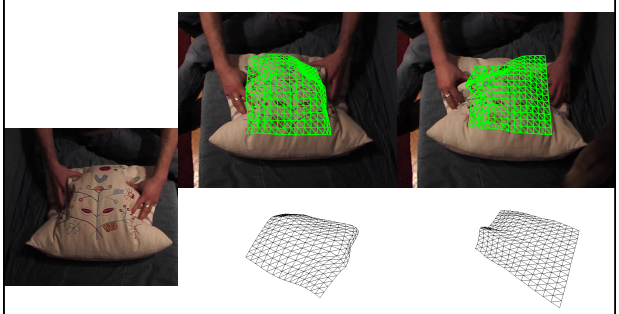


Comparison



Ground Truth Mesh
Reconstruction by our method
Reconstruction by method of Salzmann et al

Cushion



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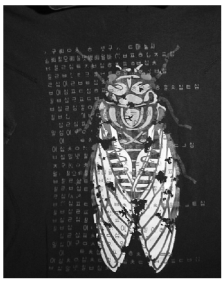
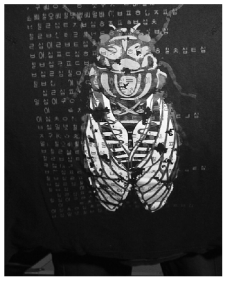
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.
- Potential applications.

Problem Formulation



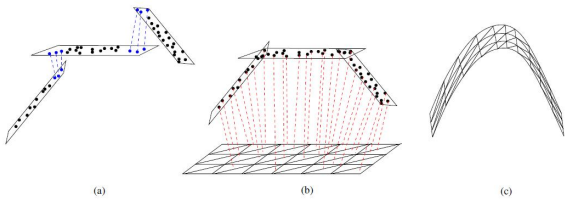
Input Frame



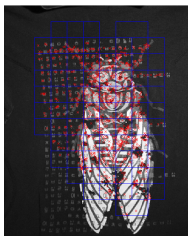
Support Frame

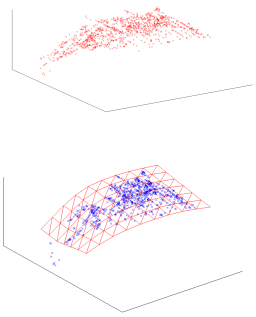
Varol et al., ICCV 2009

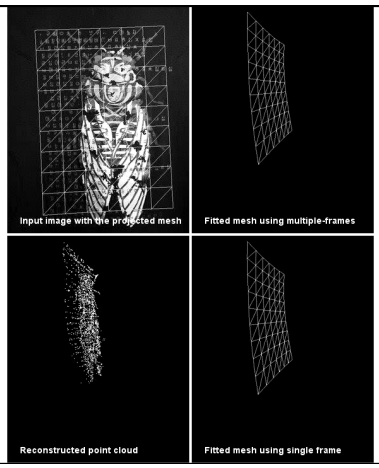
Local Homographies

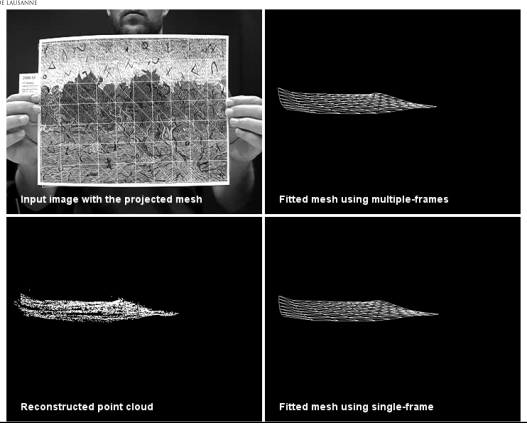


Consistent Point Cloud









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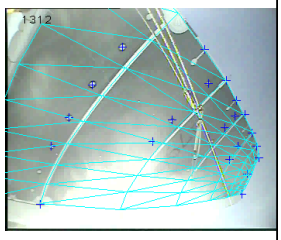
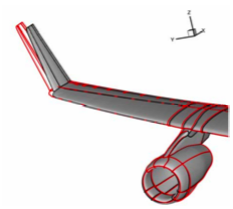
Hydroptère



Runs at 8 Hz on an ordinary PC

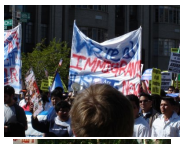
Wing Deformation



- Compare predicted and observed values.
- Improve simulation software until the two match.

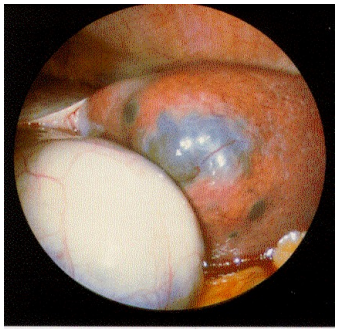
--> Virtual wind tunnel.

Intelligence Gathering



- Automated reading of those banners requires unwarping the surfaces.

Laparoscopic Surgery



A Generic Paradigm

Automated 3D deformable surface detection:

- Reconstruct textured parts of a surface.
- Learn a deformation model from those.
- Apply it to reconstruct the rest of the surface.

→ A robust method that is easy to deploy.

Thanks to

- R. Hartley
- S. Ilic
- V. Lepetit
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- J. Pilet
- M. Salzmann
- A. Shaji
- E. Tola
- R. Urtasun
- A. Varol